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LOW-ASPECT WING IN THE LIMITED FLOW OF A NONVISCOUS FLUID, (U)
JUL 78 V I KHOLYAVKO, Y F USIK

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OF A NONVISCOUS FLUID

by

V. I. Kholyavko and Yu. F. Usik



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EDITED TRANSLATION

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Block	Italic	Transliteration	Block	Italic	Transliteration
А а	А а	A, a	Р р	Р р	R, r
Б б	Б б	B, b	С с	С с	S, s
В в	В в	V, v	Т т	Т т	T, t
Г г	Г г	G, g	У у	У у	U, u
Д д	Д д	D, d	Ф ф	Ф ф	F, f
Е е	Е е	Ye, ye; E, e*	Х х	Х х	Kh, kh
Ж ж	Ж ж	Zh, zh	Ц ц	Ц ц	Ts, ts
З э	З э	Z, z	Ч ч	Ч ч	Ch, ch
И и	И и	I, i	Ш ш	Ш ш	Sh, sh
Й й	Й й	Y, y	Щ щ	Щ щ	Shch, shch
К к	К к	K, k	Ъ ъ	Ъ ъ	"
Л л	Л л	L, l	Ы ы	Ы ы	Y, y
М м	М м	M, m	Ь ь	Ь ь	'
Н н	Н н	N, n	Э э	Э э	E, e
О о	О о	O, o	Ю ю	Ю ю	Yu, yu
П п	П п	P, p	Я я	Я я	Ya, ya

*ye initially, after vowels, and after ъ, ъ; e elsewhere.
When written as ë in Russian, transliterate as yë or ë.

RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English	Russian	English	Russian	English
sin	sin	sh	sinh	arc sh	sinh ⁻¹
cos	cos	ch	cosh	arc ch	cosh ⁻¹
tg	tan	th	tanh	arc th	tanh ⁻¹
ctg	cot	cth	coth	arc cth	coth ⁻¹
sec	sec	sch	sech	arc sch	sech ⁻¹
cosec	csc	csch	csch	arc csch	csch ⁻¹

Russian English

rot curl
lg log

LOW-ASPECT WING IN THE LIMITED FLOW OF A NONVISCOUS FLUID

V. I. Kholyavko, Yu. F. Usik

Aerodynamic characteristics are determined for a thin plane wing, moving near a solid or a free surface (hydrofoil). In this case the theory of a thin body is used. According to it the spatial flow around a thin body which is extended in the direction of movement is replaced roughly by a two-dimensional flow in transverse planes. In the case under consideration the task is reduced to the study of the movement of a plane plate near an interface (Figure 1).

Assume a plane low-aspect wing, the maximum span of which coincides with the trailing edge, is moving at a small angle of attack α with a constant velocity V_∞ . During the time dt a sector of the wing $dx_1 = V_\infty dt$ passes through the fixed transverse plane ZOY. In the equivalent two-dimensional problem this movement of the wing corresponds to the vertical shifting of a plane plate, moving with a constant velocity $V_0 = V_\infty \alpha$, by a magnitude $V_0 dt$ and a change in the width of the plate from $l(x_1)$ to

$$l(x_1) + \frac{dl(x_1)}{dx_1} dx_1 = l(x_1) + \frac{dl(x_1)}{dx_1} V_0 dt,$$

where $l(x_1)$ - local wing semispan.

Flow in the plane ZOY, caused by the vertical shifting of the

plate and a change of its width, leads to a change in the associated mass of the plate by a magnitude, corresponding to the increase in the lift of the wing on the length dx_1 , i.e.,

$$\frac{dY}{dx_1} = \frac{d(mV_0)}{dt} = V_0 \frac{dm(x_1)}{dt} = V_0 \frac{dm(x_1)}{dx_1} \frac{dx_1}{dt} = V_0^2 \alpha \frac{dm(x_1)}{dx_1}.$$

Integrating this expression for the length of the chord $0 \leq x_1 \leq b$, we obtain the lift of the wing

$$Y = \int_0^b \frac{dY}{dx_1} dx_1 = V_0^2 \alpha m(b), \quad (1)$$

where $m(b)$ - associated mass of the plate, determined in a section of the wing based on maximum span.

The magnitude of the transverse aerodynamic moment relative to the axis OZ_1 , passing through the vertex of the wing, is calculated by the formula

$$M_{x_1} = - \int_0^b x_1 \frac{dY}{dx_1} dx_1 = - V_0^2 \alpha \int_0^b x_1 dm(x_1) = - V_0^2 \alpha \times \\ \times [m(b)b - \int_0^b m(x_1) dz_1]. \quad (2)$$

The resisting force of pressure (force of inductive resistance) of the wing taking into account the drawing in effect on the leading edges is determined from the correlation

$$X_l = \frac{1}{2} Y \alpha. \quad (3)$$

We will introduce into the consideration, in place of forces and moments (1) - (3), the corresponding coefficients:

$$c_y = \frac{Y}{q_\infty S}; \\ c_{x_l} = \frac{X_l}{q_\infty S}; \quad m_{x_1} = \frac{M_{x_1}}{q_\infty S b}.$$

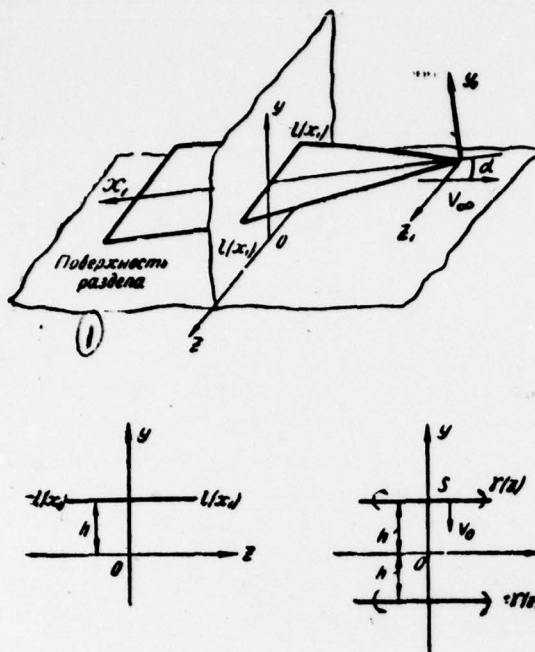


Figure 1.

Key: (1) Interface.

where S - wing area in a plane;

$$S = 2 \int_0^l l(x_1) dx_1;$$

q_∞ - dynamic head;

$$q_\infty = \frac{\rho V_\infty^2}{2}.$$

Then the aerodynamic characteristics of a low-aspect wing can be presented in such a form:

$$c_y = 2 \frac{m(b)}{\rho S} \alpha; \quad c_{x_1} = -\frac{1}{2} c_y \alpha = -\frac{m(b)}{\rho S} \alpha^2;$$

$$m_{x_1} = -2 \frac{m(b)}{\rho S} \alpha \left[1 - \frac{1}{bm(b)} \int_0^b m(x_1) dx_1 \right]. \quad (4)$$

If the coefficients of lift and lateral moment are known, then the position of the center of pressure of the wing relative to its vertex in shares of root chord can be calculated using the formula

$$x_A = -\frac{m_{z_1}}{c_y} = 1 - \frac{1}{bm(b)} \int_0^b m(x_1) dx_1. \quad (5)$$

Formulas (4) and (5) are obtained from the general dependences of the theory of a thin body and are suitable in all cases when the assumptions of this theory are valid. Let us note two special features which follow from formulas (4) and (5).

1. According to (4) the coefficients of lift and moment are a linear function of the angle of attack. The results of experimental investigations confirm the linear nature of these dependences for low aspect wings up to $\alpha \leq 6^\circ$.

2. The coefficient of lift does not depend on the shape of the wing in the plane $z_1 = l(x_1)$ and is determined only by the cross section of the wing on a section of the rear edge. The experimental data and calculations using precise theories confirm this special feature also for wings with an aspect ratio $\lambda \leq 1.5$.

Thus, in spite of the limitedness of the case $\lambda \rightarrow 0$, the theory of a thin body leads to qualitatively correct results for wings with a finite aspect ratio. It can be assumed that under certain conditions this theory will also give satisfactory quantitative results.

It ensues from (3) and (4) that the problem of determining the aerodynamic characteristics of a low-aspect wing in a limited flow is reduced to finding the associated mass of the plate near the interface. For calculation of the associated mass we will use the method of singularities and we will distribute over the length of the plate within the limits of $-1 \leq z \leq 1$ an upper layer with a continuous intensity $\gamma(z)$. It is evident that in this case $\gamma(z) = -\gamma(-z)$. We will take into account the influence of the interface using the method of mirror reflection (Figure 1).

The distribution of $\gamma(z)$ should satisfy the following condition on the plate: the particle of liquid, adjacent to the plate in any point S, should possess a constant vertical velocity V_0 (Figure 1). Using this boundary condition, we obtain an integral equation for determination of the unknown function $\gamma(z)$:

$$\int_{-1}^1 \frac{\gamma(z) dz}{z - z_1} \pm \int_{-1}^1 \frac{\gamma(z)(z - z_1) dz}{(z - z_1)^2 + 4h^2} = 2\pi V_0. \quad (6)$$

Here the "plus" sign refers to the movement of a wing under a free surface (hydrofoil), and the "minus" sign - to the movement of a wing near a solid surface (ground). Values l and h are determined in the lateral plane of flow ZOY depending on the coordinate x_1 , which enters into equation (6) as a parameter. For the calculations of the coefficients of lift and inductive resistance of a wing the values of l and h should be determined in the section of maximum span.

We will introduce a new variable ϑ from the correlation $z = l \cos \vartheta$ (when $-1 \leq z \leq 1$ we have $\pi < \vartheta < 0$) and make the substitution $\gamma(z) = \gamma(l \cos \vartheta) = \gamma(\vartheta)$, having designated $\bar{h} = h/b$. Then equation (6) is written as:

$$\int_0^{\pi} \frac{\gamma(\vartheta) \sin \vartheta d\vartheta}{\cos \vartheta - \cos \vartheta_1} \pm \int_0^{\pi} \frac{\gamma(\vartheta) (\cos \vartheta - \cos \vartheta_1) \sin \vartheta}{(\cos \vartheta - \cos \vartheta_1)^2 + 4\bar{h}^2} d\vartheta = 2\pi V_0. \quad (7)$$

Let us note the partial solution of equations (6) and (7), which corresponds to movement of a wing in a limitless fluid ($h=\infty$). From (6) and (7) with $h=\infty$ we obtain

$$\int_{-1}^1 \frac{\gamma(z) dz}{z - z_1} = \int_0^{\pi} \frac{\gamma(\vartheta) \sin \vartheta}{\cos \vartheta - \cos \vartheta_1} d\vartheta = 2\pi V_0,$$

from which

$$\gamma = 2V_0 \frac{z}{\sqrt{1-z^2}} = 2V_0 \operatorname{ctg} \vartheta. \quad (8)$$

For the solution of equation (7) in a general form with $h \neq \infty$ we present the unknown function $\gamma(\vartheta)$ as a series

$$\gamma(\vartheta) = 2V_0 [A_0 \operatorname{ctg} \vartheta + \sum_{n=1}^{\infty} A_{2n} \sin 2n \vartheta]. \quad (9)$$

in which the first term when $A_0=1$ is determined by the solution of (8), and the second term and $A_0 \neq 1$ characterize the additional load, developing on the plate from the influence of the flow boundaries.

It is evident that when $h \rightarrow \infty$ the coefficient $A_0 \rightarrow 1$, and $A_{2n} \rightarrow 0$ ($n=1, 2, \dots$).

In the writing of series (9) the condition $\gamma(\vartheta) = -\gamma(\pi - \vartheta)$ was used.

If the solution of (9) is known, then the associated mass is calculated in the following manner. With an accuracy to constant, which subsequently is not essential, the values of the function of the velocity potentials are determined on the surface of the plate when $V_0=1$:

$$\varphi_{\pm} = \pm \frac{1}{2} \int \gamma(z) dz = \mp \frac{1}{2} \int \gamma(\vartheta) \sin \vartheta d\vartheta,$$

where the upper sign refers to the upper surface (φ_{\pm}), and the lower - to the lower (φ_{\mp}). The general formula for determination of the associated mass has the form [1]

$$m = -\rho \int \varphi \frac{\partial \varphi}{\partial n} dS.$$

Since in this case on the lower surface of the plate $\frac{\partial \varphi}{\partial n} = +1$, and on the upper $\frac{\partial \varphi}{\partial n} = -1$, and furthermore $dS=dz$, then

$$\begin{aligned} m &= -4\rho \int_0^l \varphi_{\pm}(z) dz = -4\rho l \int_0^{\frac{\pi}{2}} \varphi_{\pm}(\vartheta) \sin \vartheta d\vartheta = \\ &= 2\rho l^2 \int_0^{\frac{\pi}{2}} \sin \vartheta d\vartheta \int \gamma(\vartheta) \sin \vartheta d\vartheta. \end{aligned}$$

Substituting into this formula the expression $\gamma(\vartheta)$ from (9), we obtain

$$m = \pi \rho l^2 \left(A_0 + \frac{A_2}{2} \right). \quad (10)$$

Thus for determination of the associated mass of the plate it is necessary to know the first two coefficients A_0 and A_2 of expansion (9). The remaining coefficients A_{2n} ($n=2, 3, 4, \dots$) characterize the distribution of load on the span of the plate and do not have a direct influence on the total aerodynamic characteristics of the wing.

When $h=\infty$ from the solution of (8) it follows that $A_0=1$ and $A_{2n}=0$, therefore according to (10)

$$m_\infty = \pi \rho l^2. \quad (11)$$

The aerodynamic characteristics of a wing with a calculation of (11) are determined using formulas (4) and (5)

$$C_{y_\infty} = \frac{\pi \lambda}{2} \alpha; \quad C_{x_l} = \frac{1}{2} C_{y_\infty} \alpha = \frac{1}{4} C_{y_\infty}^2; \\ x_\lambda = 1 - \frac{1}{b l^2(b)} \int_0^b l^2(x_1) dx_1. \quad (12)$$

Here $\lambda = \frac{4l^2(b)}{S}$ - aspect ratio of the wing

$z_1=l(x_1)$ - equation for the form of the wing in a plane.

Formulas (12) are the known correlations of a low-aspect wing in a limitless fluid [2].

We will switch to the calculation of coefficients of expansion A_{2n} ($n=0, 1, 2, \dots$). Equation (7) with a calculation of (9) is transformed as:

$$A_0 - \sum_{n=1}^{\infty} A_{2n} \cos 2n \theta_s \pm I(\bar{h}, \theta_s) = 1, \quad (13)$$

where the following designations are introduced

$$I(\bar{h}, \theta_s) = A_0 \left(\frac{1}{2} J_0 - \cos \theta_s J_1 + \frac{1}{2} J_2 \right) - \\ - \frac{1}{2} \cos \theta_s \sum_{n=1}^{\infty} A_{2n} (J_{2n-1} + J_{2n+1}) + \frac{1}{4} \sum_{n=1}^{\infty} A_{2n} (J_{2n-2} - J_{2n+2}); \\ J_m = \frac{1}{\pi} \int_0^\pi \frac{\cos m \theta d\theta}{(\cos \theta - \cos \theta_s)^2 + 4\bar{h}^2}.$$

After the calculation of integrals J_m and further conversions of formula (13) we finally obtain the equations for the coefficients of expansion A_{2n} ($n=0, 1, 2, \dots$).

1. For movement of a wing near a solid surface

$$A_0 \zeta_0(\bar{h}, \theta_s) + 1 = \sum_{n=1}^{\infty} A_{2n} [\zeta_{2n}(\bar{h}, \theta_s) - \cos 2n \theta_s]. \quad (14)$$

2. For movement of a wing under a free surface (hydrofoil)

$$A_0 [2 + \zeta_0(\bar{h}, \vartheta_0)] - 1 = \sum_{n=1}^{\infty} A_{2n} [\zeta_{2n}(\bar{h}, \vartheta_0) - \cos 2n \vartheta_0]. \quad (15)$$

The following designations are introduced in (14) and (15):

$$\begin{aligned} \zeta_0(\bar{h}, \vartheta_0) &= -\frac{u}{V} |\cos \vartheta_0| - 2\bar{h} \frac{u}{V}; \\ \zeta_{2n}(\bar{h}, \vartheta_0) &= \left(\frac{u + \cos \vartheta_0}{t} \right)^{2n} T_{2n}(t); \\ u &= \frac{1}{2} V \sqrt{V^2 - (\sin^2 \vartheta_0 + 4\bar{h}^2)}; \\ \bar{u} &= \frac{1}{2} V \sqrt{V^2 + \sin^2 \vartheta_0 + 4\bar{h}^2}; \\ V &= \sqrt{(\sin^2 \vartheta_0 + 4\bar{h}^2)^2 + 16\bar{h}^2 \cos^2 \vartheta_0}; \\ t &= \frac{u}{\sqrt{u^2 + 4\bar{h}^2}}; \quad T_{2n}(t) = \cos(2n \arccos t), \end{aligned}$$

where $T_{2n}(t)$ - Chebyshev polynomial of I family.

If in correlations (14) or (15) a series of values of ϑ_0 is fixed, then we will obtain a system of algebraic equations for the determination of the coefficients of expansion A_{2n} ($n=0, 1, \dots$).

In this case the number of fixed points ϑ_{s1} ($i=1, 2, \dots$) determines the number of equations in systems (14) and (15) and, consequently, the number of unknown coefficients A_{2n} ($n=0, 1, 2, \dots, i-1$). The remaining coefficients ($n=i, i+1, \dots$) should be accepted as equal to zero.

Using (14) and (15) calculations were made for the coefficients in the case of fixing of one point (I approximation), three (II approximation) and five (III approximation).

Part of the calculations for movement of a wing near a solid surface are presented in Figure 2. Based on the known coefficients A_0 and A_2 and formula (10) it is possible to find the associated mass of a plate, and with formulas (4) and (5) - the aerodynamic characteristics of the wing.

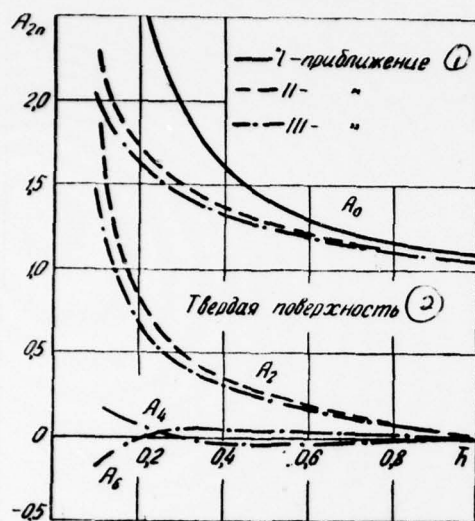


Figure 2.

Key: (1) -approximation; (2) Solid surface.

Figure 3 gives the magnitudes of the derivative of the coefficient of lift based on the angle of attack C_y^a , related to the value of $C_{y\infty}^a$ in limitless flow (12). Here also the dots designate the average values of \bar{C}_y^a , obtained using the theory of a vortical carrying surface for rectangular wings with $\lambda=0.4-1.6$ [4].

Analysis of the calculations made shows that satisfactory results right up to $\bar{h} \gg 0.1$ can be obtained already in the III approximation for a wing close to a solid surface, and in II - for a hydrofoil.

If we are limited by the values $\bar{h} \gg 0.4$ (wing close to the ground) and $\bar{h} \gg 0.25$ (hydrofoil), then the aerodynamic characteristics of a low-aspect wing can be determined using the I approximation. In this case we have simple analytical dependences, which follow from (14) and (15) when $\vartheta_s = \pi/2$ and $A_{2n}=0$ ($n=1, 2, 3\dots$).

1. For a wing close to a solid surface

$$A_0 = \frac{\sqrt{1+4\bar{h}^2}}{2\bar{h}}, \quad m = \pi \rho l^2 \frac{\sqrt{1+4\bar{h}^2}}{2\bar{h}}.$$

Since

$$\bar{C}_v^a = \frac{m}{m_\infty},$$

then

$$\bar{C}_v^a = \frac{\sqrt{1+4\bar{h}^2}}{2\bar{h}}.$$

2. For a hydrofoil

$$A_0 = \frac{1}{2 - \frac{2\bar{h}}{\sqrt{1+4\bar{h}^2}}}; \quad m = \pi \rho l^2 \frac{1}{2 - \frac{2\bar{h}}{\sqrt{1+4\bar{h}^2}}}; \quad (17)$$

$$\bar{C}_v^a = \frac{1}{2 - \frac{2\bar{h}}{\sqrt{1+4\bar{h}^2}}}.$$

Calculations using formulas (16) and (17) are shown by the solid lines.

Let us note that if in expression $\frac{\sqrt{1+(2\bar{h})^2}}{2\bar{h}}$ we replace 2 by a larger number (3 for example), then formulas (16) and (17) give satisfactory coincidence with calculations using the III approximation up to $\bar{h} \gg 0.10$.

Thus it is possible to recommend the following approximation formulas for calculation of the associated mass of a plate in a limited flow when $\bar{h} \gg 0.10$.

1. Near a solid surface

$$m = \pi \rho \frac{\sqrt{1+9\bar{h}^2}}{3\bar{h}} l^2. \quad (18)$$

2. Under a free surface

$$m = \frac{\pi \rho l^2}{2 - \frac{3\bar{h}}{\sqrt{1+9\bar{h}^2}}}. \quad (19)$$

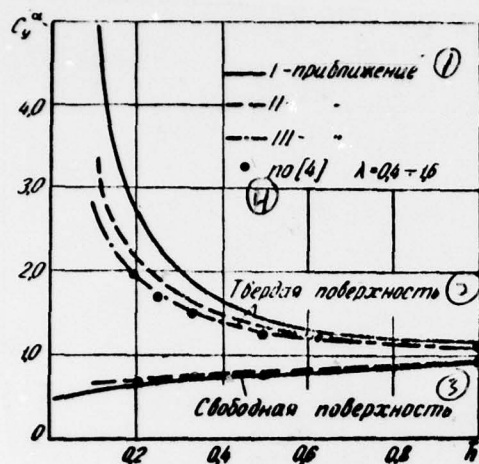


Figure 3.

Key: (1) -approximation; (2) Solid surface; (3) Free surface; (4) according to [4].

Formulas (18) and (19) are suitable for the calculations of moment characteristics and the position of the center of pressure of wings. For example, for a wing close to a solid surface from (5) and (18) we find

$$x_0 = 1 - \frac{\bar{h}}{bl^2(b) \sqrt{1 + 9\bar{h}^2}} \int_0^1 l^2(x_1) \sqrt{\frac{l^2(x_1) + 9\bar{h}^2(x_1)}{h(x_1)}} dx_1.$$

Before the integral $\bar{h} = h/l(b)$, h - position of the rear edge of the wing relative to the interface, $l(b)$ - wing semispan.

In the integrand expression the magnitudes of h and l depend on the coordinate x_1 . At small angles of attack

$$h(x_1) = h + (b - x_1)\alpha = l(b) \left[\bar{h} + \frac{b}{l(b)} (1 - x_1)\alpha \right].$$

In the assumptions of the theory of a thin body $\frac{b}{l(b)} \sim \frac{1}{\alpha}$ and the second term in the right side under certain conditions can be of the order of the first term, therefore the position of the center of pressure on a plane wing close to an interface depends on the angle of attack.

We will be limited to the case $\alpha \rightarrow 0$, when $h(x_1) = l(b)\bar{h} = \text{const}$ and the position of the center of pressure is determined by the formula

$$x_0 = 1 - \frac{l}{V_{1+9\bar{h}^2}} \int_0^l \bar{l}^3(x_1) \sqrt{\bar{l}^2(x_1) + 9\bar{h}^2} dx_1, \quad (20)$$

where $\bar{l}(x_1) = \frac{l(x_1)}{l(b)}$ - relative local wing semispan; $\bar{x}_1 = \frac{x_1}{b}$.

Let us consider a family of wings, in which the shape in the plane is assigned by the equation $\bar{l}(x_1) = \bar{x}_1^m$.

The exponent m changes within the limits $0 < m < \infty$. When $m=1$ we obtain a delta wing, when $m \rightarrow 0$ - rectangular.

According to formula (20) we will have

$$x_0 = 1 - \frac{1}{V_{1+9\bar{h}^2}} \int_0^1 \bar{x}_1^{2m} \sqrt{\bar{x}_1^{2m} + 9\bar{h}^2} d\bar{x}_1. \quad (21)$$

If similar transformations are fulfilled for a hydrofoil, then from (5) and (19) when $\alpha \rightarrow 0$ it follows that

$$x_0 = 1 - \left(2 - \frac{3\bar{h}}{V_{1+9\bar{h}^2}} \right) \int_0^1 \frac{\bar{l}^3(x_1) \sqrt{\bar{l}^2(x_1) + 9\bar{h}^2}}{2V_{\bar{l}^2(x_1) + 9\bar{h}^2 - 3\bar{h}}} d\bar{x}_1. \quad (22)$$

For a family of wings $\bar{l}(\bar{x}_1) = \bar{x}_1^m$

$$x_0 = 1 - \left(2 - \frac{3\bar{h}}{V_{1+9\bar{h}^2}} \right) \int_0^1 \frac{\bar{x}_1^{2m} \sqrt{\bar{x}_1^{2m} + 9\bar{h}^2}}{2V_{\bar{x}_1^{2m} + 9\bar{h}^2 - 3\bar{h}}} d\bar{x}_1. \quad (23)$$

When $\bar{h} = \infty$ (flow of a limitless fluid) from (21) and (23)

$$x_{0\infty} = \frac{2m}{2m+1}. \quad (24)$$

This result can also be determined from formula (12).

In a general case the integrals in formulas (21) and (23) are expressed by hypergeometric functions [3]. Figure 4 gives the

calculations of $\bar{x}_0 = \frac{x_0}{x_{0\infty}}$ for wings with $m = 1/2, 1, 2$ (solid lines - for a wing close to a solid surface, broken - for a hydrofoil).

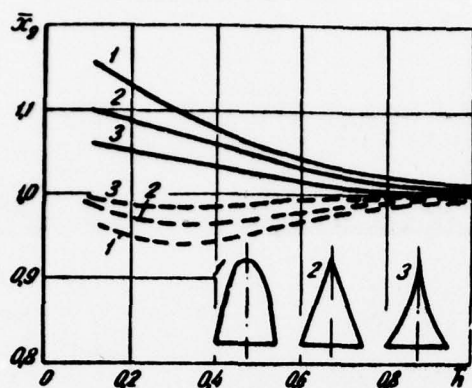


Figure 4.

An analysis of the results obtained shows that with the approximation of a wing to a solid surface (ground) the center of pressure is shifted to the trailing edge, and this shift is greater, the smaller the exponent m . In particular, for a rectangular wing ($m \rightarrow 0$) the greatest shift should be observed.

The influence of the boundary of a free surface for a hydrofoil is opposite to the effect of the ground.

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C043 USAMIIA	1	E404 AEDC	1
C509 BALLISTIC RES LABS	1	E408 AFWL	1
C510 AIR MOBILITY R&D	1	E410 ADTC	1
LAB/FIO		E413 ESD	2
C513 PICATINNY ARSENAL	1	FTD	
C535 AVIATION SYS COMD	1	CCN	1
C591 FSTC	5	ASD/FTD/NICD	3
C619 MIA REDSTONE	1	NIA/PHS	1
D008 NISC	1	NICD	2
H300 USAICE (USAREUR)	1		
P005 ERDA	1		
P005 CIA/CRS/ADB/SD	1		
NAVORDSTA (50L)	1		
NASA/KSI	1		
AFIT/LD	1		